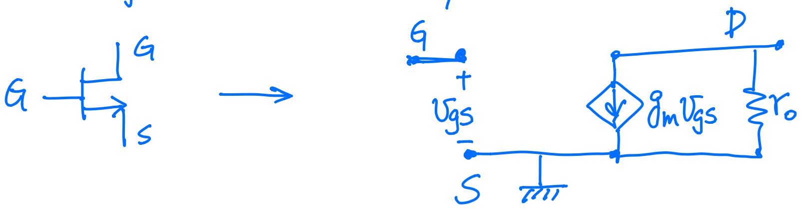
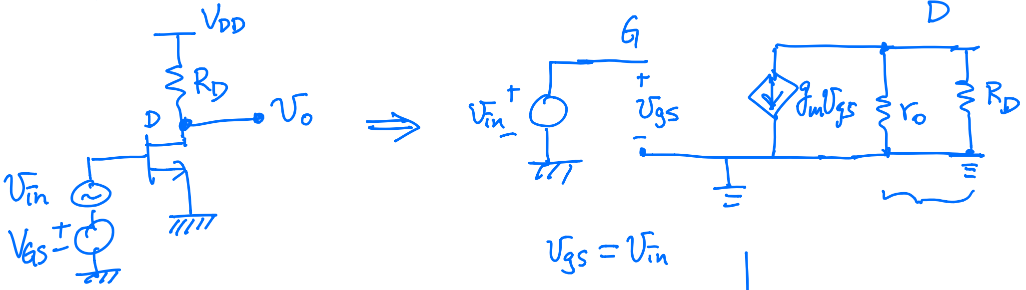
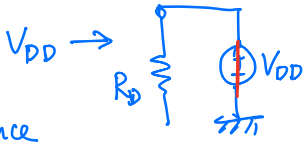
Small-Signal Model : Hybrid π

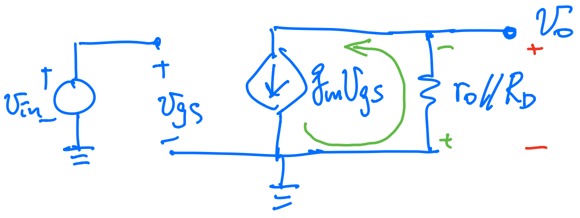


Example



$v_{gs} = v_{in}$





Transconductance

$$g_m = \frac{\partial i_D}{\partial v_{gs}} = k_n \cdot V_{OV}$$

$[S] = \left[\frac{mA}{V}\right]$ $\quad$ $\left[\frac{mA}{V^2}\right][V]$

$[\mho]$

$$r_o = \frac{1}{\frac{\partial i_D}{\partial v_{DS}}} \Rightarrow \left[\frac{V}{A}\right] = [\Omega]$$

want this as large as possible

Think $r_o \sim 100k\Omega$

$$v_o = -(g_m v_{gs})\cdot(r_o // R_D)$$

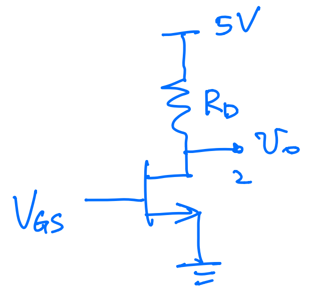
$$v_{in} = v_{gs}$$

$$A_v = \frac{v_o}{v_{in}} = -g_m(r_o // R_D)$$

$[S] \times [\Omega] = [1]$

If $R_D = 5k\Omega$ $\quad r_o // R_D \approx R_D$

Design Example



Assume $k_n = 1\,mA/V^2$, $k_{t,n} = 1V$

Goal : obtain $|A_v| = 10$

$|A_v| = g_m \cdot (r_o // R_D) \approx g_m \underset{\uparrow}{R_D} = 10$

$r_o \gg R_D$

Choose $I_D = 2mA \Rightarrow g_m = k_n V_{ov} = 2\ mS$

$I_D = \frac{1}{2} k_n \cdot V_{ov}^2 \Rightarrow V_{ov} = 2V$

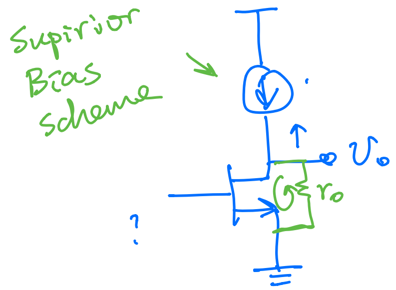
$V_{DS,min} = V_{GS} - V_t = V_{ov} = 2V$

Choose $V_o = 3.5V$

$R_D = \frac{5-3.5}{2mA} = \frac{1.5V}{2mA} = 0.75\,k\Omega$

$A_v = 2\,mS \times 0.75\,k\Omega = 1.5$

Alternative Design



$k_n = 1\,mA/V^2$. $V_{t,n} = 1V$ $\lambda \rightarrow r_o = 100k\Omega$

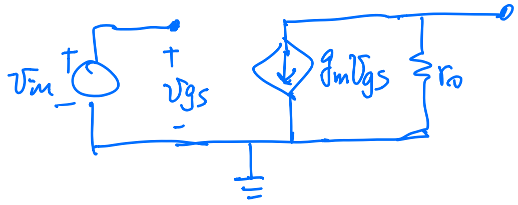
$g_m = k_n V_{ov}$ → Express g_m in terms of I_D

$I_D = \frac{1}{2} k_n V_{ov}^2 \Rightarrow \sqrt{2 I_D k_n} = \sqrt{k_n^2 V_{ov}^2} = k_n V_{ov}$

$g_m = \sqrt{2 I_D k_n}$

$v_o = -(g_m v_{gs}) \cdot r_o \Rightarrow \underset{\uparrow}{A_v} = -g_m r_o$

Intrinsic voltage gain of a transistor



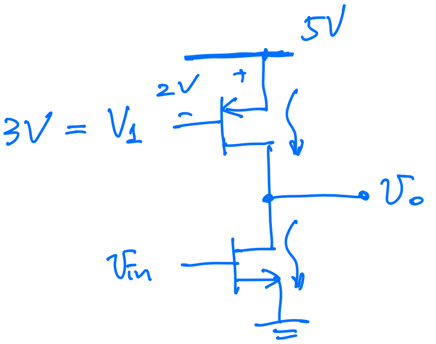
$|A_v| = g_m \cdot 100k\Omega = 10$

$g_m = \frac{10}{10^5} = 10^{-4}\ S$

$10^{-4} = \sqrt{2 I_D k_n} \Rightarrow 10^{-8} = 2 I_D \overset{10^{-3}}{\overset{\downarrow}{k_n}}$

$I_D = \frac{1}{2} \times 10^{-5} = 5\,\mu A$

$\Rightarrow V_{ov}, \quad \Rightarrow V_{GS}$



$k_n = k_p = 1\,mA/V^2$

$V_{t,p} = -1V$

$$I_D = I_{D,p} = I_{D,n}$$

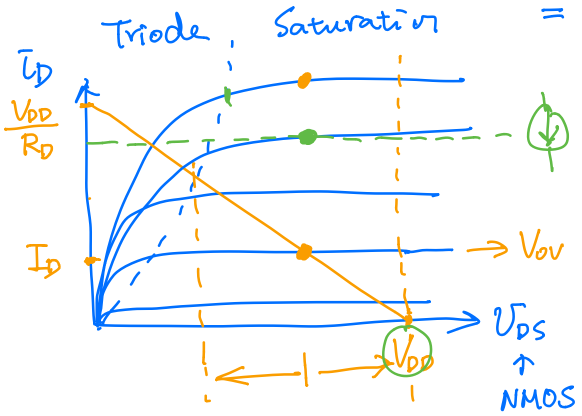
$$= \frac{1}{2} k_p |V_{ov,p}|^2$$

Want $I_D = 0.5mA$

$$|V_{ov,p}| = \frac{2I_D}{k_p} = 1\,V = \underset{2}{|V_{GS,p}|} - \underset{1}{|V_{t,p}|}$$

$$\underline{g_m = \sqrt{2 I_D k_n}} = \sqrt{2 \times 0.5\,mA \cdot 1\,mA/V^2}$$

$$= 1\,mA/V = 1\,mS$$



Recap Small-signal analysis

① Solve DC bias problem

② Calculate. g_m, r_o

③ Draw AC equivalent circuit

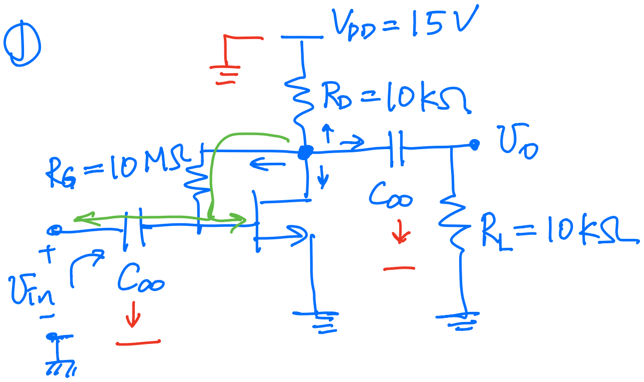
→ Replace independent voltage source → short ckt

" current → open

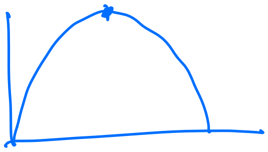
" MOS → Hybrid π

④ Do KCL. KVL, solve v_o

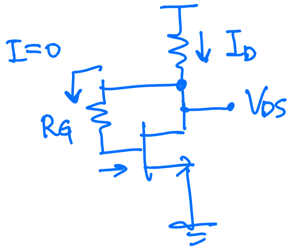
①



$$\begin{cases} k_n = 0.25\ mA/V^2 \\ V_{tn} = 1.5V \\ \lambda = \frac{1}{50V} \end{cases}$$



DC → $C_\infty \equiv$ Open ckt



$V_{GS} = V_{DS}$

⇓

(i) $I_D = \frac{1}{2}k_n V_{ov}^2 = \frac{1}{2}k_n(V_{GS}-V_{tn})^2 = \frac{1}{2}k_n(V_{DS}-V_{tn})^2$

KVL:

(ii) $V_{DD} = I_D R_D + V_{DS}$

⇒ Quadratic equation for V_{DS}

↳ 2 solutions

Only one is physical

$V_{DS} = 4.4V$

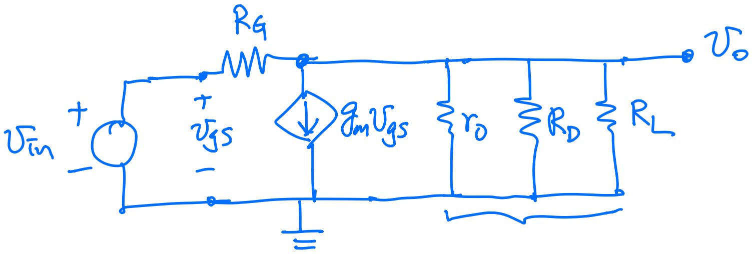
⇒ $I_D = 1.06\,mA$

Check KVL indeed satisfied

② $g_m = k_n V_{ov} = (0.25\,mA/V^2)(4.4-1.5)V = 0.725\,mA/V = 0.725\,mS$

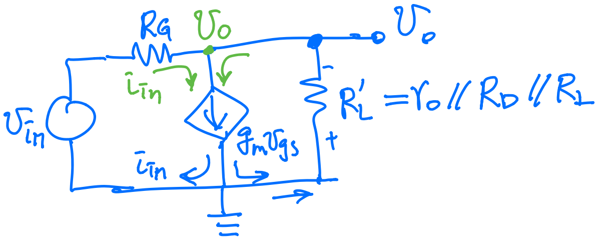
$r_o = \frac{1}{\lambda I_D} = 47\,k\Omega$

③



$v_{gs} = v_{in}$

⇓



$$i_{in} = \frac{v_{in} - v_o}{R_G}$$

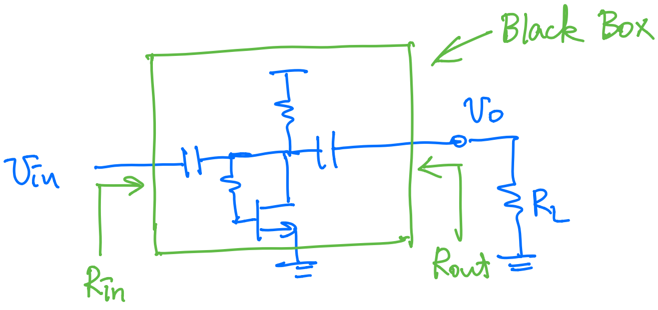
$$v_o = -(g_m \underbrace{v_{gs}}_{v_{in}} - i_{in}) \cdot R_L'$$

$$A_v = \frac{v_o}{v_{in}} = -g_m R_L' \frac{1 - \cancel{\frac{1}{g_m R_G}}}{1 + \cancel{\frac{R_L'}{R_G}}}$$

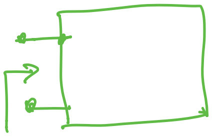
$$R_L' = \underset{47k\Omega}{r_o} // \underbrace{\underset{10k\Omega}{R_D} // \underset{10k\Omega}{R_L}}_{5k\Omega} \simeq 4.5 k\Omega$$

$g_m = 0.725$ mS. $R_G = 10 M\Omega$.

$$A_v \approx -g_m R_L' = -3.3 \text{ V/V}$$



16A/B



Thevenin equivalent ckt.

Input Resistance R_{in} of the "Whole" amplifier

- Replace DC source with short
- current " open
- C_∞ → short

* Include load, R_L